

Orthogonal experimental study on high frequency cascade thermoacoustic engine

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Abstract

Orthogonal experiment design and variance analysis were adopted to investigate a miniature cascade thermoacoustic engine, which consisted of one standing wave stage and one traveling wave stage in series, operating at about 470 Hz, using helium as the working gas. Optimum matching of the heater powers between stages was very important for the performance of a cascade thermoacoustic engine, which was obtained from the orthogonal experiments. The orthogonal experiment design considered three experimental factors, i.e. the charging pressure and the heater powers in the two stages, which varied on five different levels, respectively. According to the range analysis and variance analysis from the orthogonal experiments, the charging pressure was the most sensitive factor influencing the dynamic pressure amplitude and onset temperature. The total efficiency and the dynamic pressure amplitude increased when the traveling wave stage heater power increased. The optimum ratio of the heater powers between the traveling wave stage and the standing wave stage was about 1.25, compromising the total efficiency with the dynamic pressure amplitude.

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1. Introduction

As a fluctuating form of mechanical energy, acoustical energy can be converted into thermal energy or vice versa by thermoacoustic devices [1]. Such machines can be used to generate electricity or to provide refrigeration and air conditioning. Compared with traditional heat engines, thermoacoustic devices have some advantages as follows. Firstly, thermoacoustic devices avoid oil seals or lubricants because acoustic networks are adopted to eliminate the reciprocating moving parts in traditional heat engines. Secondly, thermoacoustic devices can be driven by low quality heat energy such as exhaust heat, solar energy etc., so it is significant for remote rural areas where electricity is unavailable. Finally, due to using inert gases as working fluid, such simple structured machines are environmentally

friendly. As a result of more than two decades of attempts, thermoacoustic conversion efficiencies have been improved to levels that rival what can be obtained from internal combustion engines[2].

Some thermoacoustic devices have been successfully applied on space shuttles and navy warships, but more extensive commercial applications are still restricted by the power density and actual efficiency [3]. Recent developments in thermoacoustics are attempting to solve this issue, such as the appearance of hybrid thermoacoustic devices [4–7]. The thermoacoustic conversion efficiency is mostly determined by the laws of thermodynamic cycles that the working fluid particle experiences in thermoacoustic cores, namely, regenerators or stacks that provide solid heat capacity. According to the acoustic field characteristic, such particle thermodynamic cycles are of two essential types, Brayton cycles and Stirling cycles [8]. The former is employed in standing wave acoustic fields, for which efficiency is limited by inherent thermodynamic irreversibility,

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i.e. obvious irreversible entropy production caused by irreversible heat exchange. The latter is employed in traveling wave acoustic fields, which is as efficient as Carnot cycles in theory. A wide variety of standing wave thermoacoustic engines [9–11] have been built, but the best standing wave engines have 50% lower efficiency than traditional Stirling engines due to the deliberately imperfect thermal contact in order to introduce a significant time delay between the fluid oscillation and the fluid thermal expansion/contraction. Ceperley suggested a class of loop acoustic topology to build a pure traveling wave thermoacoustic engine [12], but there is serious viscous loss because of the low acoustic impedance (which means high oscillation velocity) in a pure traveling wave field so the actual efficiency of this kind of engine is still not high.

To enjoy the best features of the high acoustic impedance in a standing wave field and more efficient thermodynamics cycle in a traveling wave field, some hybrid thermoacoustic devices are being developed. One of these hybrid devices is the thermoacoustic Stirling hybrid engine, consisting of a looped tube jointing with a branch resonator, yielded 42% of the Carnot efficiency [2,4]. Unfortunately, this kind of device brings a circulating second order mass flow, called “Gedeon streaming” (because Gedeon wrote the first clear paper on this subject) [13], that can decrease the efficiency advantage by convecting heat between the hot and cool heat exchangers. It is often difficult to suppress Gedeon streaming around the torus, although that may be stopped by exploiting the time averaged pressure gradient developed in the oscillating flow through an asymmetric channel [4], which consumes acoustic power and adds complexity that is undesirable in commercial devices. In addition, the thermal expansion of the hot parts in the pressure vessel bring difficulties in fabrication and reliability.

Another newly discovered hybrid thermoacoustic device is the cascade combination of standing wave and traveling wave engines, which creates a traveling wave acoustic network with high impedance so as to realize high efficiency thermoacoustic conversion [5–7]. In an appropriately designed acoustic resonator, such as a half wave length waveguide tube with both end cavities, the acoustic field consists of a partial traveling wave component in the center of the resonator and a standing wave component in the other region. The so called cascade thermoacoustic devices are composed of stack units in the standing wave region and regenerator units in the traveling wave region. Because of the straight line topology, Gedeon streaming is easy to eliminate. The power density and efficiency are all high because more acoustic power is amplified in the regenerator units. The pressure oscillation is excited from the stack units and then amplified in the regenerator units. So, the coupling between the stack units and the regenerator units will affect the cascade system performance, such as the resonance frequency, the amplitude and phase of the dynamic pressure, the onset temperature and the total efficiency. Only very few studies on cascade thermoacoustic systems

have been performed, being ongoing in Los Alamos National Laboratory and the Technical Institute of Physics and Chemistry, CAS. The study in Los Alamos tested a three stage cascade thermoacoustic engine operating at 25 Hz, which was a pioneering work but ignored the interaction effect between stages. To reveal the coupling characteristic of the cascade thermoacoustic device and to improve its performance, some detailed studies have been developed in our laboratory. In this paper, the orthogonal experimental study on the matching between stages will be described. A straight forward extension of this work can also contribute to the investigation of other coupling systems with multi-thermoacoustic cores, which is being the increasing direction in the thermoacoustic field. For example, the early investigation of a two stack annular thermoacoustic prime mover provided by Hsiao-Tseng Lin et al. described the system eigen-modes but paid little attention to the interaction among stages [10,11]. Besides the stages’ coupling effect, the charging pressure in a cascade system has an important influence on the running mode because it changes in impedance and depth of penetration. To study this class of multi-level system with multi-influencing factors, hundreds of experiments would often be required. As an effective and economical experimental method, the orthogonal experimental method was adopted to reduce the number of experiments. Using the normalized orthogonal tables to arrange experiments, the experimental times were decreased greatly and the veracity and reliability of the experimental results were ensured because the selected representative experimental points obey statistical laws [14]. According to the range analysis on the orthogonal experiments, the primary and lesser influencing factors in the cascade system can be detected. The influencing distinctiveness of the different experimental factors can also be distinguished by the variance analysis. The optimum matching between stages’ heater powers was obtained.

2. Experimental apparatus

As shown in Fig. 1, the experimental prototype was a 1.0 m-long mini-type cascade thermoacoustic engine composed of one standing wave stage and one traveling wave stage, which operated at 470 Hz using helium as the working gas [6,7]. These two stages were arranged in series on the center of the half wave resonator that had only one dynamic pressure anti-node at the navel. The resonator was a half wave waveguide tube with two cavities in its terminals. The standing wave engine consisted of the stack sandwiched with the ambient heat exchanger (AHX) and hot heat exchanger (HHX). The stack was structured with stacked 60 mesh stainless steel screens. The AHX was a plate type heat exchanger, with helium gas oscillating in the spaces of the plates and ambient temperature water flowing in the round jacket spaces surrounding the fins. The HHXs consisted of ceramic honeycombs through which the long electric heating filament was wound. The traveling wave engine had a similar configuration, in which

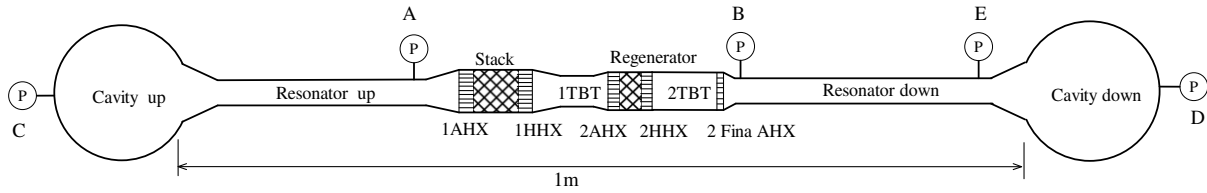


Fig. 1. The schematic depiction of the high frequency cascade thermoacoustic engine experimental apparatus. Where AHX was the ambient heat exchanger; HHX was the heater heat exchanger; TBT was the thermal buffer tube; *P* was the pressure sensor; A, B, C, D and E represented different sensors' location.

the sandwiched regenerator was filled with 200 mesh steel stainless screens. In addition, the thermal buffer tubes (TBT) connected the two stages smoothly to insulate the AHX from the adjacent HHX.

In order to measure the hybrid operating mode of the standing wave and traveling wave, five piezoelectric pressure sensors were arranged along the resonator, denoted in Fig. 1 using capital letters. Sensor A is located in front of the standing wave engine, and sensor B is located behind the traveling wave engine. Sensor C and D, located on the up cavity and down cavity, respectively, were near the oscillation velocity anti-node. Sensor E is located near the entry to the down cavity, which was close to the dynamic pressure node. The three middle sensors, A, B and E are distributed in equal intervals. All these sensors' signals were amplified via a charge amplifier and then transmitted to a lock in amplifier, which recorded the resonant frequency and the values of the dynamic pressure amplitude and phase with a 1% relative error. The six thermocouples (not shown in Fig. 1) were located on the bottom, the center and the top of the stack and regenerator, respectively, and the other two thermocouples were set on both extreme cavities. The thermoelectricity signals from these thermocouples were read and recorded by a digital multimeter, which brought $\pm 1^\circ\text{C}$ measurement accuracy. All the dynamic signals of the pressure sensors and thermocouples were also obtained by a data acquisition system, comprised of data acquisition cards, a computer and a self program, editing in VB. The two stages' heater powers were supplied by two DC power sources separately and measured by reading the values of the volts and currents of the DC source, which caused $\pm 0.5\text{ W}$ measured error. Helium of high purity was adopted as the working gas.

3. Orthogonal experiment design

One of the aims of the orthogonal experiments was to find the optimum matching of the heater powers between the two stages. The second aim was to detect the most important experimental factor on the performance of the cascade systems, especially on the onset temperature and the dynamic pressure amplitude. The last aim was to investigate how each experimental factor influenced the operating frequency, the onset temperature and the dynamic pressure amplitude. Because there were extensive variety

span for each experimental factor and various compounding modes among them, the reasonable and condensed levels of each factor should be chosen before the orthogonal experiments design.

Because the thermal efficiency (the ratio of acoustic power to heater power) was about 20% and 30% for the standing wave engine and the traveling wave engine, respectively, the total efficiency of the cascade thermoacoustic engine was determined by the proportion of the heater powers in the different stages. When more heater power was input from the traveling wave stage, higher total efficiency would be obtained. However, the heater power in the standing wave stage should also have enough heater power to overcome the viscous loss in the regenerator and the stack until the oscillation was excited.

The acoustic power producing rate in the standing wave stage was written as Eq. (1) approximately, where E_2 was the acoustic power; x was the axial coordinate along the axial length of stack; p_1 was the dynamic pressure in the stack; ω was the operating angular frequency; A was the flow area of the working gas; γ was the ratio of isobaric to isochoric specific heats of the working gas; f_v, f_k were the viscous and thermal relaxation spatial average complex geometry factors of the stack; dT_m/dx was the temperature gradient of the working gas in the stack; and ∇T_{crit} was the critical temperature gradient, estimated as $\nabla T_{\text{crit}} = \frac{|p_1|/\rho_m C_p}{|U_1|/A\omega}$, where U_1 was the volume flow rate in oscillation; ρ_m was the mean density of the working gas; and C_p was the isobaric heat capacity per unit mass [8].

$$\frac{dE_2}{dx} = \frac{1}{2} |p_1|^2 \omega A \frac{\gamma - 1}{\gamma p_m} \times \text{Im}(-f_k) \left(\frac{dT_m/dx}{\nabla T_{\text{crit}}} - 1 \right) \quad (1)$$

$$\Delta E_2 = E_{20} \left(\frac{T_H}{T_C} - 1 \right) - \frac{3\mu\Delta x}{2Ar_H^2} |U_1|^2 - \frac{\omega^2 Ar_H^2}{6kT_m} |p_1|^2 \quad (2)$$

According to Eq. (1), the acoustic power was increased with the increase of the temperature gradient in the stack. The temperature gradient was also proportionate to the heater power, so the acoustic power produced in the stack was correlative with the heater power. Further more, the amplitude of the dynamic pressure would be higher if more acoustic power were produced. The heater power should be kept on a certain level to attain sufficiently high dynamic pressure amplitude.

The amplified acoustic power in the regenerator was described in Eq. (2), where the latter two terms were vis-

cous and thermal relaxation loss, respectively; E_{20} was the acoustic power introduced to the regenerator from the standing wave stage; T_H and T_C were the temperature on the hot end and cold end of the regenerator, respectively; μ was the dynamic viscosity of the working gas; Δx was the length of the regenerator; r_H was the hydrodynamic radius of the regenerator's channels; and k was the thermal conductivity of the working gas. According to Eq. (2), the amplified acoustic power was directly proportional to the temperature gradient and the acoustic power delivered from the standing wave stage. To attain more acoustic power output, the heater power in the standing wave stage should not be low, and the traveling wave stage should input as much heater power as possible.

On the other hand, reducing the onset temperature of thermoacoustic systems was the key technique to utilize low quality heat energy. The onset temperature has the affinity with the part of the heater power that exceeded the threshold heater power [15], so the matching of the heater powers between stages could determine the onset condition and the oscillation mode to a certain extent, which then influenced the dynamic pressure amplitude and the acoustic power.

Based on the above analysis and the primary experimental experience, the heater power in the standing wave stage (noted as Q_1) was divided into five levels, i.e. 40 W, 80 W, 100 W, 120 W and 130 W. The heater power in the traveling wave stage (noted as Q_2) was also divided into five levels, which were 20 W, 60 W, 110 W, 140 W and 150 W. To study carefully how the charging pressure affects the onset temperature, resonance frequency and the dynamic amplitude, the charging pressure (noted as p_m) was separated into five levels as 1.2 MPa, 1.4 MPa, 1.6 MPa, 1.8 MPa and 2.0 MPa. After the experimental factors and corresponding varied levels had been determined, the orthogonal table (noted as $L_{25}(5^3)$) could then be chosen as the experimental scheme, which would denote the different assorted experimental programs and would greatly decrease the times of experiments, shown as Table 1, where 25 was the total number of experiments, 5 was the number

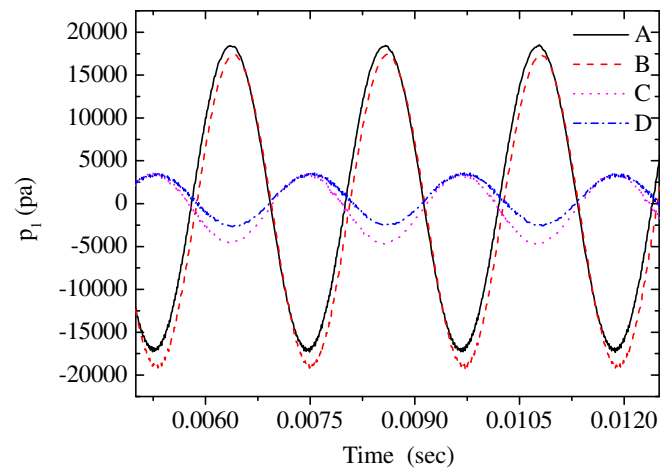


Fig. 2. The stable dynamic pressure waveform at different locations for a typical experiment. Where A, B, C, D denoted different pressure sensors that were shown in Fig. 1.

of levels and 3 was the factors number [14]. Each experiment followed the same experimental procedure. Initially, adjusting the charging and exhausting gas system set a certain level of charging pressure according to the orthogonal table. Then, tuning the DC sources heated the two hot heat exchangers to certain levels until oscillations appeared. The instantaneous onset temperatures in the stack and the regenerator were all recorded in a computer. Along with the hot temperatures' increase, the dynamic pressure amplitude increased. At last, stable temperatures and dynamic pressures were obtained. During all the process, the dynamic pressure signals were also acquired by a computer or a lock in amplifier. For example, Figs. 2 and 3 show temporal and spatial stable dynamic pressure fields, respectively. In each experiment, satisfactory agreement between the measured and calculated dynamic pressure field was obtained, for which the most relative error was not more than 8% including 1% measurement error. Fig. 3 showed the comparison between the experiment result and the simulation for one of the orthogonal experiments. These sim-

Table 1

The orthogonal experimental scheme for 3-factors with 5-levels, where the two latter rows were the measured stable temperatures (T_{H1} , T_{H2}) on the hot end of the stack and the regenerator, respectively

No.	Q_1 (W)	Q_2 (W)	P_m (MPa)	T_{H1} (K)	T_{H2} (K)	No.	Q_1 (W)	Q_2 (W)	P_m (MPa)	T_{H1} (K)	T_{H2} (K)
1	40	20	1.2	420	650	14	100	140	1.2	795	867
2	40	60	1.4	425	720	15	100	150	1.4	783	825
3	40	110	1.6	431	811	16	120	20	1.8	835	395
4	40	140	1.8	451	821	17	120	60	2.0	827	570
5	40	150	2.0	470	841	18	120	110	1.2	835	780
6	80	20	1.4	729	395	19	120	140	1.4	835	787
7	80	60	1.6	711	571	20	120	150	1.6	828	830
8	80	110	1.8	724	733	21	130	20	2.0	851	393
9	80	140	2.0	716	809	22	130	60	1.2	872	594
10	80	150	1.2	726	874	23	130	110	1.4	873	730
11	100	20	1.4	773	400	24	130	140	1.6	866	789
12	100	60	1.8	778	579	25	130	150	1.8	853	823
13	100	110	2.0	772	711						

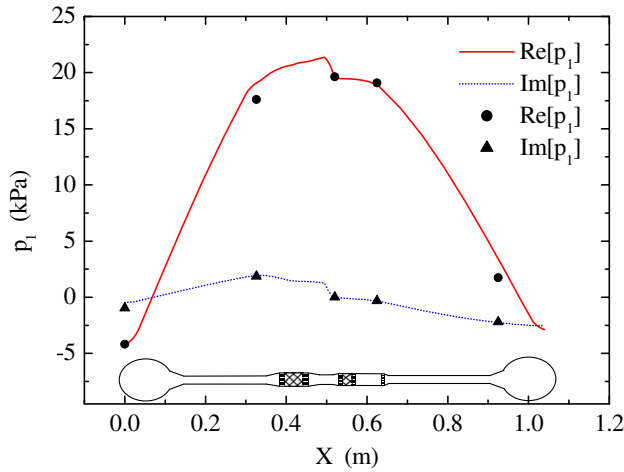


Fig. 3. Components of the dynamic pressure field for a typical experiment. Points were measured values, and lines were calculations by DeltaE. The center points phase taken to be zero and not measured.

ulations were the numerical integration of the linear acoustic equations using the DeltaE program [16].

4. Orthogonal experiment analysis

The method to process the orthogonal experimental data was very important because the experimental times had been greatly reduced. The onset temperature was a limiting factor to extending utilization of the cascade thermoacoustic engine, which was an important evaluating indicator in the orthogonal experiments. The operating frequency could alter the position of the traveling wave region and then affect the system performance, so it was another important evaluating indicator. To improve the power density, the dynamic pressure amplitude should be kept high enough, which was the third evaluating indicator. Because the orthogonal experiments were performed under different compoundings among p_m , Q_1 and Q_2 , the most sensitive factor influencing the above evaluating indicators should be detected, which was often operated by the method of range analysis. To study further on how the operating conditions were to affect the performance of the cascade thermoacoustic system, the method of variance analysis and distinctiveness verification should be adopt. According to the above methods, the optimum ratio of the heater powers between two stages could be determined.

If the experimental values at the same level in the orthogonal experiments for each experimental factor were added together, the weighting table for each level would be attained. According to the weighting table, the optimum compounding among p_m , Q_1 and Q_2 could be found, which revealed the various trends for the different experimental factors and then guided the further experiments with more optimum experimental compounding modes. The level current graph, a kind of broken line graph with set levels as the abscissa and the corresponding weighting results as the ordinate, could filter the optimum experimental corre-

Table 2

The range analysis on the dynamic pressure experimental results, where the Roman numerals represented the weighting value on different level number; Roman numerals with down-lineation denoted the average weighting value on the according level; j denoted a certain factor; R was the range

Factors	Q_1	Q_2	p_m
I_j	0.00	3.06	1.56
II_j	2.84	2.98	2.74
III_j	3.50	3.04	3.09
IV_j	4.19	2.94	3.64
V_j	4.48	3.00	4.00
I_j	0.00	0.76	0.39
II_j	0.56	0.74	0.68
III_j	0.70	0.76	0.77
IV_j	0.83	0.73	0.91
V_j	0.89	0.75	0.99
R	0.32	0.03	0.60

sponding mode that was often missed in orthogonal experiment design.

For example, in the processing of the experimental results about the dynamic pressure amplitude relative to the charging pressure, the sum of each experimental factor at the same level was calculated and noted as I_j , II_j , III_j , IV_j , V_j , where the Roman numerals represent the level number and j denotes a certain factor. The average value at the same level for each experimental factor was also calculated, noted as the Roman numerals of the column shown in Table 2. The level current graph was drawn from Table 2, which was shown in Figs. 4–6. From these figures, the onset temperature and the dynamic pressure amplitude were all increased, and the operating frequency had a little drop when the charging pressure was increased. Along with the Q_1 increasing, the dynamic pressure amplitude was increased and the frequency had a little rise, but the onset temperature was decreased. When Q_2 was increased, the

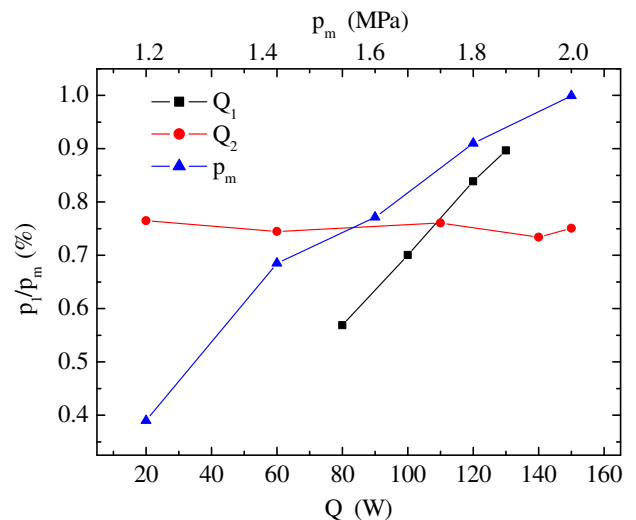


Fig. 4. The level current graph of the dynamic pressure amplitude for different experimental factors, where p_i/p_m was the ratio of the dynamic pressure amplitude measured by sensor B to the charging pressure p_m .

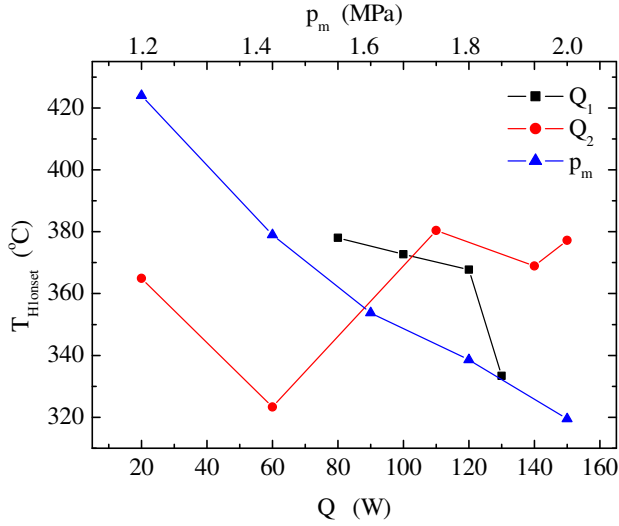


Fig. 5. The level current graph of the onset temperature for different experimental factors, where $T_{Hlonset}$ was the onset temperature on the hot end of the stack.

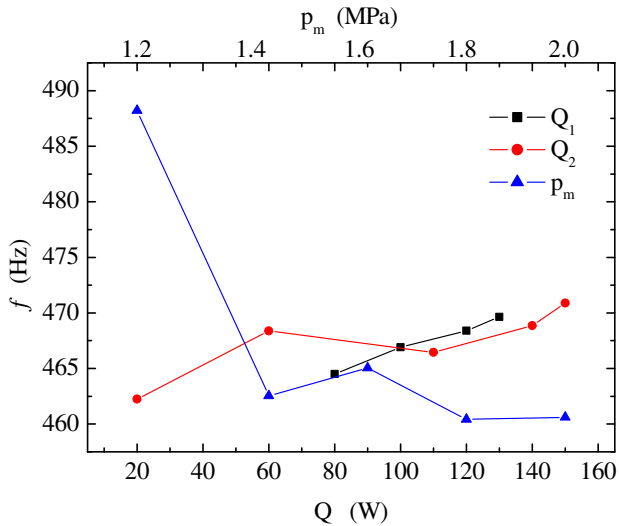


Fig. 6. The level current graph of the operating frequency for different experimental factors, where f was operating frequency.

frequency rose, and the dynamic pressure amplitude had no evident change, but the onset temperature fluctuated. Although the Q_2 had little effect on the dynamic pressure amplitude, the amplified acoustic power in the regenerator was increased with Q_2 , which means more power was created through the reversible cycles, so the system efficiency must be improved. The variety of acoustic power in the regenerator behaved as the phase change of the dynamic pressure, which could consult to the phase tuning study [17].

The range analysis could discriminate the comparative importance of each factor, which was the maximum value minus the minimum value of the experimental average value, noted as R , defined as Eq. (3), where k was the times

of the same level; and j denoted the different experimental factors.

$$R_j = \max\{I_j/k_j, II_j/k_j, \dots\} - \min\{I_j/k_j, II_j/k_j, \dots\} \quad (3)$$

By the range analysis on the dynamic pressure amplitude, the important influencing factors were p_m , Q_1 , Q_2 in sequence. According to similar analysis on the onset temperature and frequency, p_m had the most sensitive effect on the onset temperature and frequency, and Q_2 had more sensitive effect on the frequency and onset temperature than Q_1 . However, the total cascade efficiency was often estimated as Eq. (4), which would be higher if Q_2 were higher than Q_1 .

$$\eta \approx (0.20 \times Q_1 + 0.30 \times Q_2) / (Q_1 + Q_2) \quad (4)$$

Compromising between the efficiency and the dynamic pressure amplitude, the optimum experimental matching was the compounding of $Q_1 = 120$ W, $Q_2 = 150$ W and $p_m = 2.0$ MPa, for which the theoretical thermal efficiency would be 25.6%.

Further variance analysis could detect the distinctiveness of each experimental factor, which included the following steps:

Firstly, to calculate the average value of the experimental results for each factor, defined as $\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$, for example, the average ratio of the dynamic pressure amplitude to charging pressure was 0.751%.

Secondly, to calculate the sum of deviations squared and which total value, defined as Eq. (5), which was equal to the total experimental times divided by each level number.

$$S_j = k_j \left(\frac{I_j}{k_j} - \bar{y} \right)^2 + k_j \left(\frac{II_j}{k_j} - \bar{y} \right)^2 + k_j \left(\frac{III_j}{k_j} - \bar{y} \right)^2 + \dots$$

$$S_{\text{total}} = \sum_{i=1}^n (y_i - \bar{y})^2 \quad (5)$$

For example, the S_{total} was 1.28 for the relative dynamic pressure amplitude.

Thirdly, to calculate the variance for each factor, $V_j = S_j/f_j$, where f_j was the freedom defined as the level number minus one.

Fourthly, to calculate the ratio of variance, $F_j = V_j/V_e$, where the denominator was the error variance.

Finally, to verify the distinctiveness of each factor consulting to the F -distribution laws, $F(a, f_1, f_2)$, where f_1 and f_2 were the freedoms of the numerator and denominator in the ratio of variance, respectively. The consulting value of F -distribution used here included $F_{0.01}(5, 19) = 4.17$, $F_{0.01}(4, 19) = 4.5$, $F_{0.10}(4, 19) = 2.27$ [14]. According to the distinctiveness verification shown in Table 3, the most sensitive factor influencing the dynamic pressure amplitude was Q_1 and p_m . For the onset temperature, Q_2 and p_m had also high marked influence and Q_1 was only a marked factor. The operating frequency was influenced by p_m at the high marked level to a certain extent, but the heater powers had no marked influence. Compared with the additional

Table 3

Distinctiveness verification that showed the marked difference for each experimental factor

Parameters	Factors		
	Q_1	Q_2	p_m
p_1			
Variance ratio	24.1	0.3	88.2
Distinctiveness	High marked	Not marked	High marked
$T_{H1}(\text{Onset})$			
Variance ratio	3.22	5.95	18.4
Distinctiveness	Marked	High marked	High marked
f			
Variance ratio	0.17	0.51	6.71
Distinctiveness	Not marked	Not marked	High marked

experiments only operating the standing wave stage, the regenerator unit could increase the dynamic pressure amplitude by 30–50% and decrease the onset temperature about 50–100 °C, which showed the important effect on improving the performance of the cascade thermoacoustic system.

5. Conclusions

The performance of the 470 Hz cascade thermoacoustic system, especially the coupling character between different stage heater powers, has been studied based on the orthogonal experimental analysis on the dynamic pressure amplitude, the onset temperature and the operating frequency. Three factors were chosen in the orthogonal experimental design, which were p_m , Q_1 and Q_2 , and each was varied at five different levels. According to the range analysis, the important factors influencing the dynamic pressure amplitude and resonance frequency were p_m , Q_1 and Q_2 in sequence; the order of what influenced the onset temperature was p_m , Q_2 and Q_1 . The level current graph showed the variety of character when the factors changed. As the charging pressure increased, the onset temperature reduced, the dynamic pressure amplitude rose and the frequency had a little drop. When Q_1 increased, the dynamic pressure amplitude also increased, the onset temperature decreased and the frequency had a little rise. If Q_2 increased, there was no outstanding change in the dynamic pressure amplitude, and the onset temperature had some fluctuating, and the frequency increased to a certain extent. With the increase in the proportion of Q_2 relative to Q_1 , the theoretical efficiency could be improved, with ideal matching of 1.25 (Q_2/Q_1) on the high charging pressure

condition. Further variance analysis and distinctiveness verification of each factor demonstrated the influence on the cascade system performance in quantity. The charging pressure was the most sensitive factor for the dynamic pressure amplitude, the onset temperature and the operating frequency. Q_1 had more marked effect on the onset temperature than Q_2 , and Q_1 had more sensitive effect on the pressure amplitude than Q_2 .

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